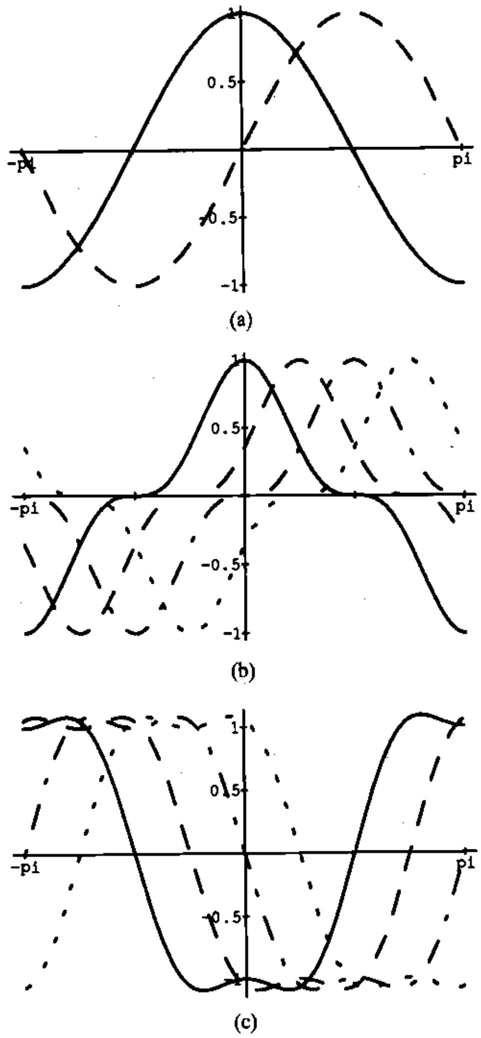

Fig. 2. Three sets of steerable basis functions, plotted as a function of azimuthal angle ϕ at a constant radius. An arbitrary angular offset of each function (linear shift, as plotted here) can be obtained by a linear combination of the basis functions shown: (a) G_1 steerable basis set; (b) four basis functions for $0.25\cos(3\phi) + 0.75\cos(\phi)$; (c) four basis functions for $0.25\cos(3\phi) - 1.25\cos(\phi)$. The same interpolation functions apply for (b) as for (c).

graph (rotation of the filter corresponds to translation on these graphs). Fig. 2(a) shows the sinusoidal variation of 1–D slices of $G_1^{0°}$ and $G_1^{90°}$ plotted at a constant radius. In this case, the steering property is a restatement of the fact that a linear combination of two sinusoids can synthesize a sinusoid of arbitrary phase. Fig. 2(b) and (c) are 1–D cross sections of steerable basis sets for functions with the azimuthal distribution $0.25\cos(3\phi) + 0.75\cos(\phi)$ and $0.25\cos(3\phi) - 1.25\cos(\phi)$, respectively. Since each function has nonzero Fourier coefficients for two frequencies, by **Theorem 1**, four basis functions suffice for steering. Because both functions contain sinusoids of the same frequencies (even though of different amplitudes), they use the same $k_j(\theta)$ interpolation coefficients.

It is convenient to have a version of **Theorem 1** for functions expressed as polynomials in Cartesian coordinates x and y [12]. In Appendix C, we prove the following theorem:

Theorem 3: Let $f(x,y) = W(r)P_N(x,y)$, where $W(r)$

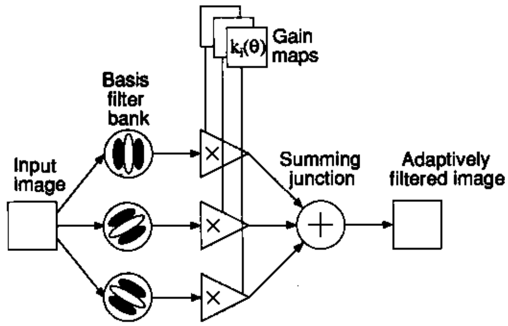

Fig. 3. Steerable filter system block diagram. A bank of dedicated filters process the image. Their outputs are multiplied by a set of gain maps that adaptively control the orientation of the synthesized filter.

is an arbitrary windowing function, and $P_N(x,y)$ is an Nth order polynomial in x and y, whose coefficients may depend on r. Linear combinations of $2N+1$ basis functions are sufficient to synthesize $f(x,y) = W(r)P_N(x,y)$ rotated to any angle. Equation (10) gives the interpolation functions $k_j(\theta)$. If $P_N(x,y)$ contains only even [odd] order terms (terms $x^n y^m$ for $n+m$ even [odd]), then $N+1$ basis functions are sufficient, and (10) can be modified to contain only the even [odd] numbered rows (counting from zero) of the left-hand side column vector and the right-hand side matrix.

Theorem 3 allows steerable filters to be designed by fitting the desired filters with polynomials times rotationally symmetric window functions, which can be simpler than using a Fourier series in polar coordinates. However, **Theorem 3** is not guaranteed to find the minimum number of basis functions that can steer a filter. Representing the function in a Fourier series in angle makes explicit the minimum number of basis filters required to steer it. In a polynomial representation, the polynomial order only indicates a number of basis functions sufficient for steering. For example, consider the angularly symmetric function $x^2 + y^2$ written in a polar representation as $r^2 e^{0\phi}$. **Theorem 2** would say that only one basis function is required to steer it; **Theorem 3**, which uses only the polynomial order, merely says that a number of basis functions sufficient for steering is $2 + 1 = 3$.

The above theorems show that steerability is a property of a wide variety of functions, namely, all functions that can be expressed as a Fourier series in angle or in a polynomial expansion in x and y times a radially symmetric window function. Derivatives of Gaussians of all orders are steerable because each one is a polynomial (the Hermite polynomials [32]) times a radially symmetric window function.

Fig. 3 shows a general architecture for using steerable filters (cf. Koenderink and van Doorn [22]–[24], who used such an architecture with derivatives of Gaussians, and Knutsson *et al.* [21], who used it with related filters). The front end consists of a bank of permanent, dedicated basis filters, which always convolve the image as it comes in; their outputs are multiplied by a set of gain masks, which apply the appropriate interpolation functions at each position and time. The final summation produces the adaptively filtered image.

An alternative approach to the steerable filters presented here would be to project all rotations of a function onto a